Leapfrog Numerical MHD Solution of Hale Bopp Comet

Salman Z. Khalaf, Ahmed A. Selman and Elaff E. Al-lateef

Abstract – We investigate the physical properties of solar wind interaction with comet Hale Bopp, taking a rectangular numerical solution of MHD Equations. Simulations were numerically made using computer codes written with MATLAB 7.8 based on a mid-point, 3-D Leapfrog explicit method. Our results excluded the pressure and energy results where it was shown that interaction near the cometary nucleus is mainly affected by the new ions added to the plasma of the solar wind, which increased the average molecular weight. When this effect was taken under careful consideration, some characteristics of the cometary tail could be driven in the presence of the IMF. The previous set of conditions used in [1] were also used in this study.

Keywords: Comet tail interactions - Hale Bopp comet - MHD - Computer Simulation.

1 INTRODUCTION

NUMERICAL solution of cometary tail interaction with solar wind was of interest in space since Biermann discovered that solar wind has a great effects on the cometary tails shape–see review of [2]. An extensive progress has been made since then in the understanding of the interaction between the solar wind and cometary tail [3]. The prediction of Alfven [4] that the IMF plays an important role in the interaction process was later supported by a well description as due to the magneto-hydrodynamic theory (MHD) [5, 6]. This assumption made possible the first simplified one dimensional hydrodynamic models of Biermann [2] which demonstrated the basic physics of cometary reactions. Later modeling has been improved by Brosowski and Wegmann, as well as Wallis, and Wegmann [7]. In 1998, K. Murawski et al. applied the numerical simulations performed in the framework of nonlinear two-dimensional magneto hydrodynamics to investigate the solar wind interaction with Comet Halley at 0.83 A.U. corresponding to the Vega 2 encounter [8]. Comet Hale-Bopp was a target for many studies [5,6. In the present paper we shall investigate the system's model using numerical leap-frog method for three dimensions Cartesian space. The behavior of the system with distance is the main parameter discussed in this paper.

2 PHYSICAL MODEL

Theory of the physical model are the same found in [1]. Here we write the set of equations only

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho\upsilon) = \dot{\rho_c} \quad (1)$$

$$\dot{\rho_c} = \frac{G\sigma m_c}{4\pi r^2 \upsilon_c} e^{-\frac{\sigma}{\upsilon_c} r} \quad (2)$$

$$\frac{\partial n}{\partial t} + \nabla.(n\upsilon) = \dot{n_c} \quad (3)$$

$$\dot{n_c} = \dot{\rho_c}/m_c \quad (4)$$

$$\frac{\partial \upsilon}{\partial t} - \upsilon\nabla\upsilon + \frac{1}{\rho}\nabla P + \frac{B}{4\pi\rho}\nabla B = 0 \quad (5)$$

$$\frac{\partial B}{\partial t} = [\nabla \times [\upsilon \times B]] \quad (6)$$

$$\frac{\partial P}{\partial t} + \upsilon\frac{\partial \rho}{\partial r} + \gamma\rho\frac{\partial \upsilon}{\partial r} = \dot{P_c} \quad (7)$$

$$e_c = \frac{1}{2}\dot{\rho_c}\dot{\upsilon_c} \quad (8)$$

where $\dot{\rho_c}$ is the mass source of cometary ions G, σ, r and υ_c are the production rate, the ionization rate, the distance from the nucleus and the Solar wind particles velocity, m_c represents the constant molecular mass with typical value $m_c = 20m_p$ (m_p is the proton mass), P, B are the pressure and the magnetic field, $\dot{P_c}$ is the pressure source term related to the internal energy source term (e_c).

3 RESULT AND DISCUSSION

In this paper three dimensional MHD system was assumed to simulate cometary plasma of Hale-Bopp comet interaction with solar wind. The main results produced in this research are the physical properties of the cometary plasma which change due to the interaction with the solar wind. This interaction is described by the main conservation laws, equations (1-8). The velocity equation has a source term but it mentioned because it contributes with ~2% to the equation thus it wasn't included in the present simulation. As for the magnetic field, there is no source term – see [1]. The instantaneous solution of this system of differential equations will result in the basic physical properties of the system which are presented in this section and discussed. The results are presented in three-dimensional figures. In each figure, the mesh is given for a fixed z-axis value (2 or 19) while changing both x- and y-axes with the physical value being calculated. Since there are four dimensions in this case (three spatial axes plus the prosperity axis), the z-axis was chosen to be fixed. During the basic calculations, the following sets of physical parameters shown in

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Table (1) were chosen in order to perform the input parameters of the system.

TABLE 1
PHYSICAL PARAMETERS USED IN THE MAIN CALCULATIONS.

Symbol	Name	Arbitrary Value
v_c	Solar wind particle velocity	10
m_c	Constant molecular mass	0.5
G (s^{-1})	Production rate	10^{28}
σ□(s^{-1})	Ionization rate	0.01
n_i, n_j, n_k	The time coordinates	20
k	Iteration number	150
γ□□	Specific heat ratio	1.4
$\Delta x, \Delta y, \Delta z$	The spatial coordinates	30
Δt	The change in iteration of time	0.001

The code solves a boundary-condition problem; hence a set of initial and boundary conditions are required. This set was chosen in order to have convergent solutions with physical meanings. The set of initial conditions were applied at the first step of the program and it is required to set-up the various matrices used during the calculations. The boundary conditions, on the other hand, were repeatedly applied during each loop of the code so that the limits of the calculated physical properties are always under control. Initial and boundary conditions used in the present research are listed in Table (2).

TABLE 2
INITIAL & BOUNDARY CONDITIONS OF THE PRESENT CODE.

Quantity	Initial value	Boundary value
Mass density, ρ	10^{-11}	1.5
Particles velocity, (u_x, v_y, w_z)	0	0.5
Magnetic field, (B_x, B_y, B_z)	10^{-3}	5.5

The run time of the code was also studied. Few sets of iteration, time step and spatial steps were chosen and the program run time was measured in order to check the best combination that balances accuracy and time consuming. These results are shown in Table (3).

TABLE 3
THE RUNTIME MEASUREMENT OF THE CODE WITH DIFFERENT SET OF VARIABLES.

Set of variables changed	Values taken	Average run time for three value of the code LeapF.m
Δx	10, 20, 30	33 sec
Δy	10, 20, 30	33 sec
Δz	10, 20, 30	33 sec
Δt	0.1, 0.01, 0.001	34 sec
n_i, n_j, n_k	10	3 sec
n_i, n_j, n_k	20	34 sec
n_i, n_j, n_k	30	171 sec

In this research it has been changed some of parameters of table (1). This was made in order to characterize programming parameters on the results while fixing the main physical parameters of the system. These cases are:

3.1 Spatial coordinates

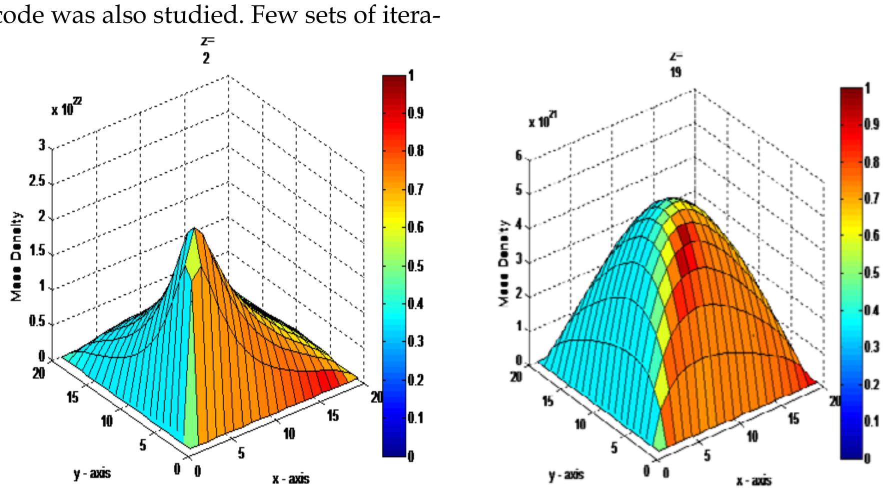
The results of the spatial coordinates are shown in Figure (1), where (Δx, Δy, Δz) were denoted by (dv).

3.2 Time coordinates

The result of changing in the time coordinates of Hale Bopp comet are shown in Figure (2), where (n_i, n_j, n_k) were denoted by N.

3.3 Number of Iteration

The result of the changing in the time iteration (dt) are show in Figure (3),



(a) Relation between mass density (amu.cm^{-3}) and space when dv=30.

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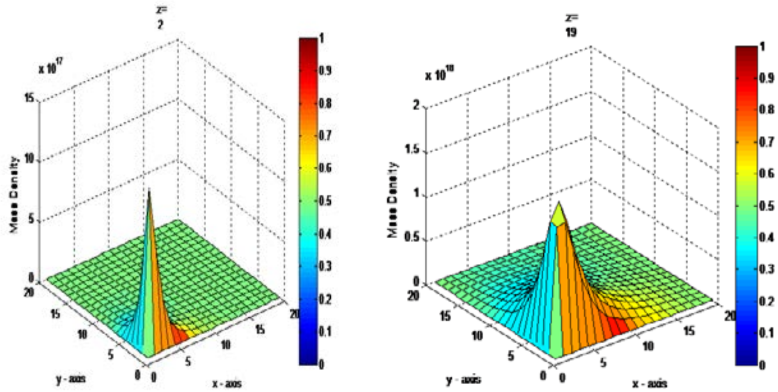
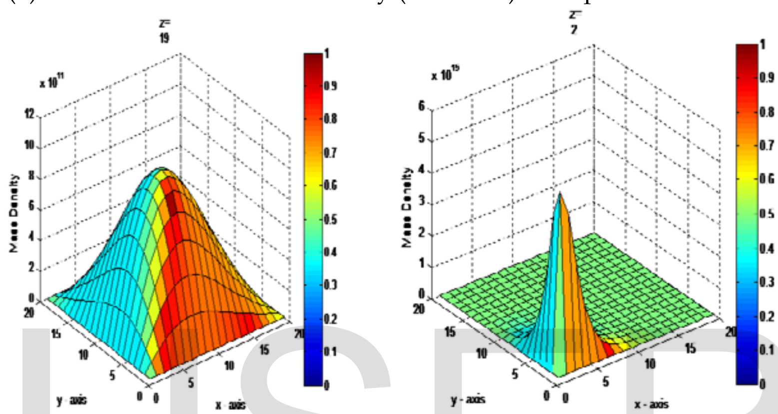


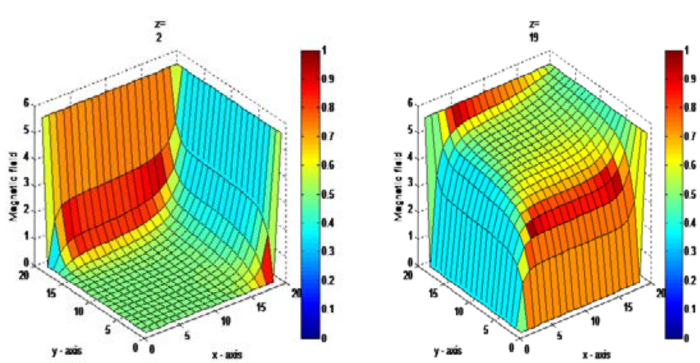
(b) Relation between mass density (amu.cm^{-3}) and space when dv=300.



(c) Relation between mass density (amu.cm^{-3}) and space when dv=3000.

Figure(1) Relation between mass density (amu.cm-3) of Hale Bopp comet for three-dimension (MHD) simulation and space (dv=30, 300, 3000), using leapfrog method.

Figure (1) shows the mass density of Hale Bopp comet with variable value of dv. In Figure (1.a) the value dv=30 was taken and the maximum value was $\simeq 1.2 \times 10^{22}$ at z=2 and become $\simeq 3.2 \times 10^{21}$ at z=19. While in Figure (1.b) the value dv=300 was taken and the maximum value was $\simeq 4 \times 10^{17}$ at z=2 and become $\simeq 0.4 \times 10^{16}$ at z=19. In Figure (1.c) the value dv=3000 was taken and the maximum was $\simeq 1.8 \times 10^{15}$ at z= 2 and become $\simeq 5.2 \times 10^{11}$ at z=19. From these it is inferred that part of the information about the mass density of Hale Bopp comet will be vanish when increasing dv. This means that a proper choice of dv must be made in order to have the best results. The mish grid in this case changes the resolution (or ability to distinguish) the comet property details. In fact the present value of dv more selected to have the best result comparing with run time and details accuracy. Figure (2) shows the magnetic field of Hale Bopp comet with variable dv value. More details can be obtained about comet with dv decreases. So the Figure (2.a) gives better the information about magnetic field of comet more than Figures (2.b) and (2.c).



(a)Relation between magnetic field (nt) and space when dv=30.

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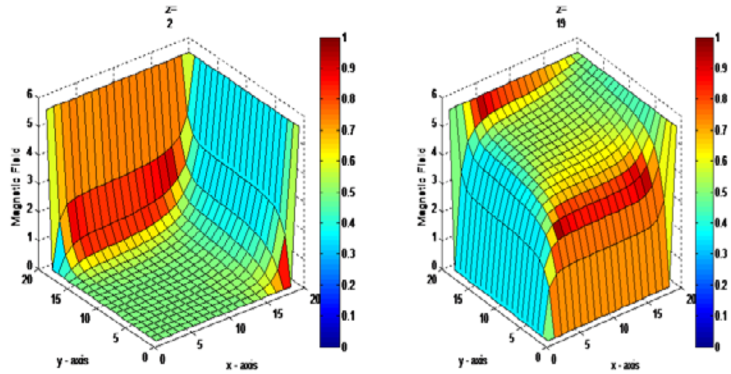
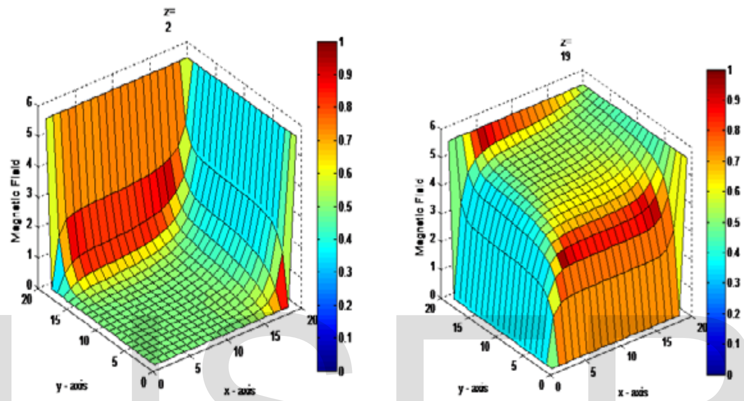
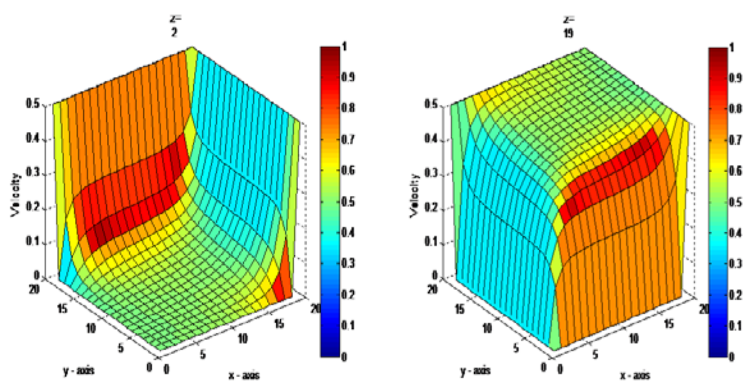


(b)Relation between magnetic field (nt) and space when dv=300.



(c) Relation between magnetic field (nt) and space when dv=3000.

Figure (2) Relation between magnetic field (nt)of Hale Bopp comet and space (dv=30, 300, 3000), using leapfrog method.



(a) relation between velocity (km.s^{-1}) and space when dv=30.

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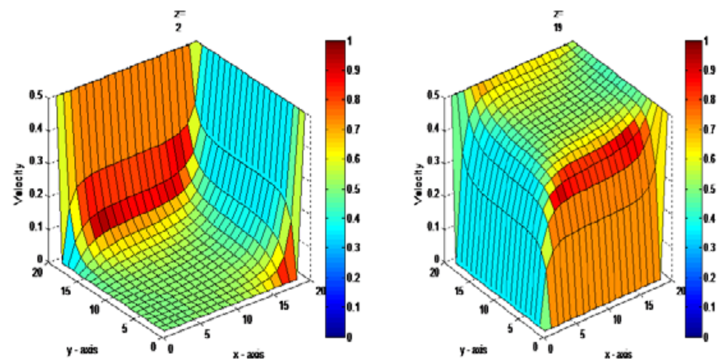
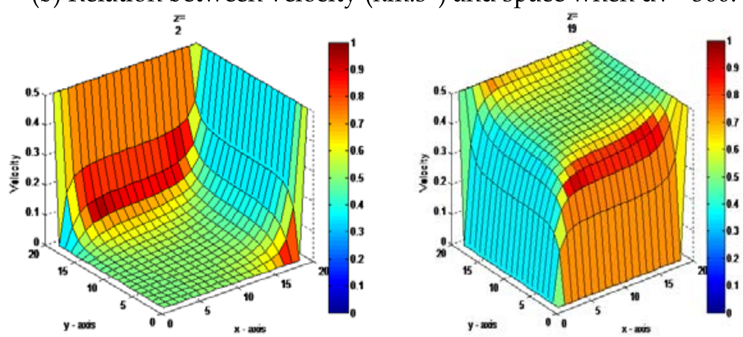


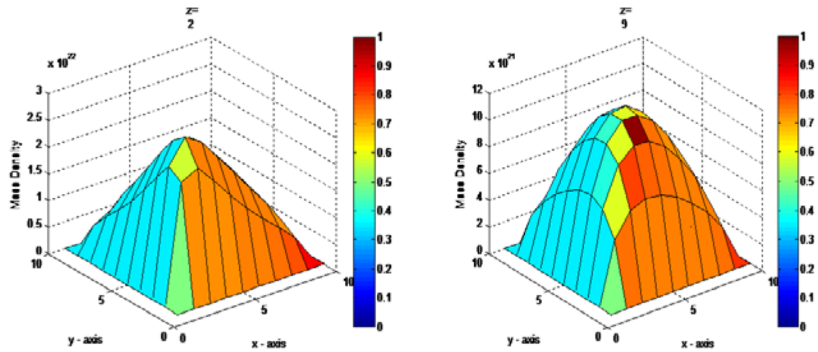
(b) Relation between velocity (km.s^{-1}) and space when dv= 300.



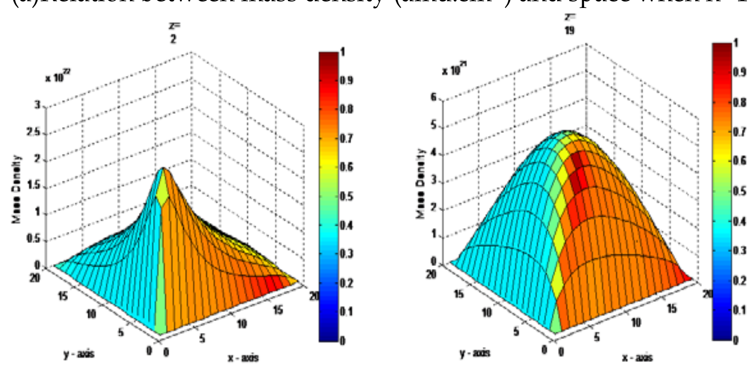
(c) Relation between velocity (km.s^{-1}) and space when dv= 3000.

Figure (3) Relation between velocity (km.s^{-1}) of Hale Bopp comet for three-dimension (MHD)simulation and space (dv=30, 300, 3000), using leapfrog method.

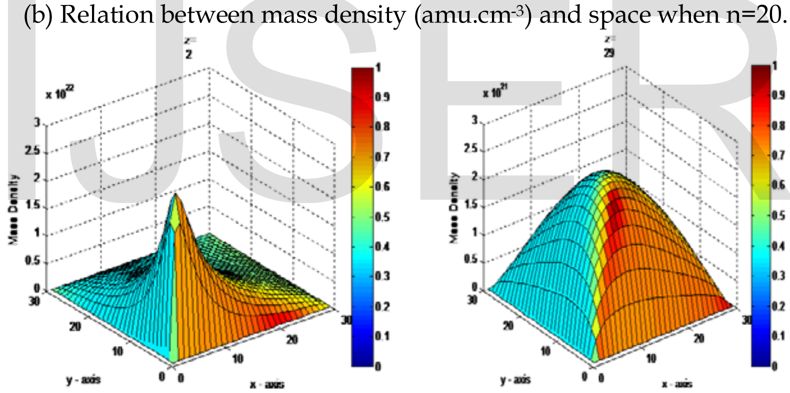
Figure (3) show the velocity of Hale Bopp comet with variable dv value. Where it is noted the part of information will be vanished as dv increases. Figure (4) shown the mass density of comet Hale Bopp with variable N value. In Figure (4.a) the value N=10 was taken and the peak was $\simeq 1.4x10^{22}$ at z=2 and become $\simeq 7.5x10^{21}$ at z=9. While in Figure (4.b) the value N=20 was taken and the peak was $\simeq 1.1x10^{22}$ at z=2 and become $\simeq 3.2x10^{21}$ at =19. And in Figure (4.c) the value N=30 was taken and the peak was $\simeq 0.9x10^{22}$at z=2 and become $\simeq 1.3x10^{21}$ at z=29.From these results one can say that, when increasing N more information can be found about mass density of comet with long time as shown in Table (3).This means that the effect was studied with longer time thus more accurate information was obtained. However, increasing time mean increasing run time of the code, as explained in the section five. From Figure (5) one can notice that when the meshgrid steps are increasing, mean that gives more details about the magnetic field of Hale Bopp comet. also, in Figure (6) has been show the velocity of Hale Bopp comet, where it has get more information by n increasing. In Figure (7) the result of mass density of Hale Bopp comet is shown with variable value of dt where in Figure (7.a) we take dt=0.1 and the peak was $\simeq 1.2x10^{22}$ at z=2 and become $\simeq 3.8x10^{21}$at z=19, while in Figure (7.b) and (c) we take dt=0.01, 0.001 respectively, and, it is found the peaks was $\simeq 1.2x10^{22}$ at z=2 and becomes $\simeq 3.2x10^{21}$ at z=19. From this it can be deduced that at (dt >0.1) there is no result, at (dt = 0.1) the result is non stable and at (dt< 0.1) the result is stable. Figure (8) represents the magnetic field of Hale Bopp comet with variable dt value. When it is noted that the Figure (8.a) is non stable result and the Figures (8.b) and (8.c) are stable results.



(a)Relation between mass density (amu.cm^{-3}) and space when n=10.

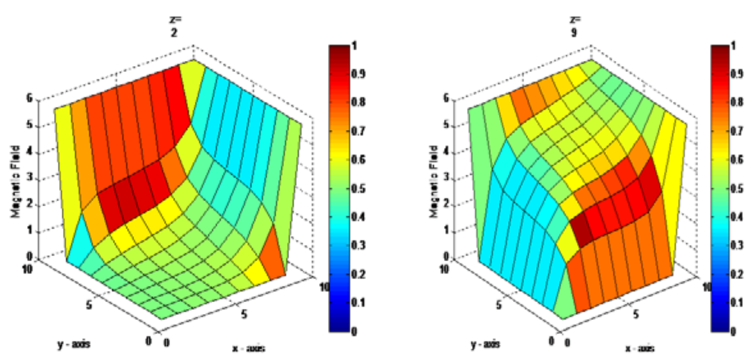


(b) Relation between mass density (amu.cm^{-3}) and space when n=20.



(c) Relation between mass density (amu.cm^{-3}) and space when n=30.

Figure (4) Relation between mass density (amu.cm^{-3}) of Hale Bopp comet and space (n=10, 20, 30), using leapfrog method.



(a) Relation between magnetic field (nt) and space when n=10.

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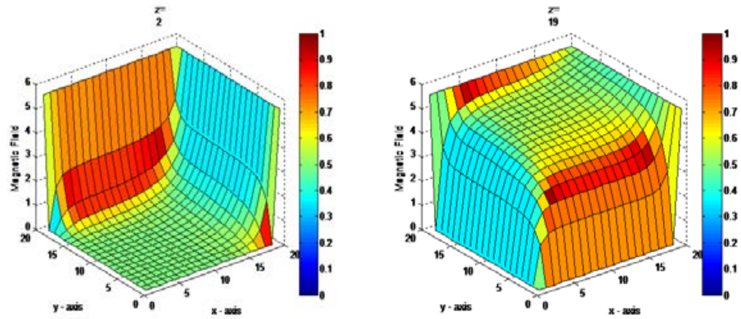
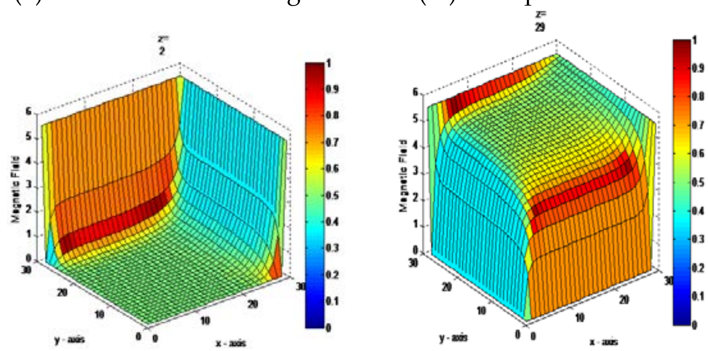


(b) Relation between magnetic field (nt) and space when n=20.



(c) Relation between magnetic field (nt) and space when n=30.

Figure(5) Relation between magnetic field (nt) of Hale Bopp comet and space (n=10,20,30), using leapfrog method.

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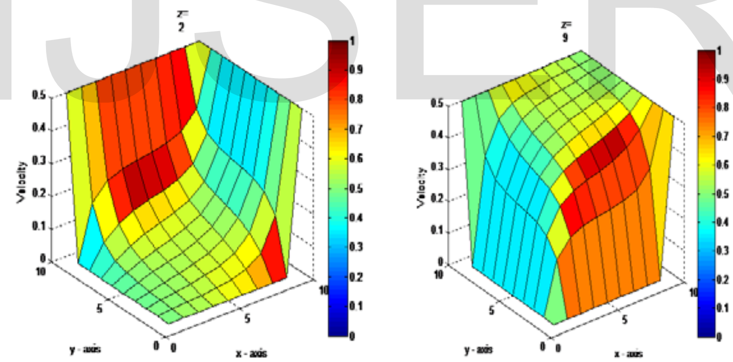
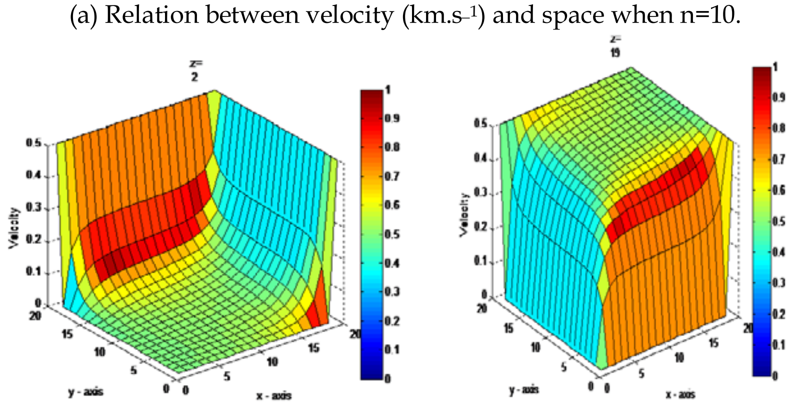
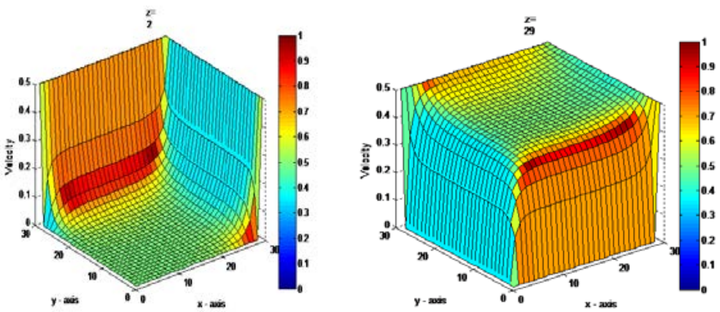


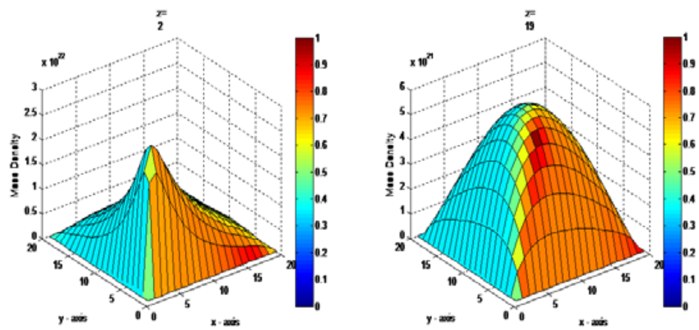
(a) Relation between velocity (km.s^{-1}) and space when n=10.



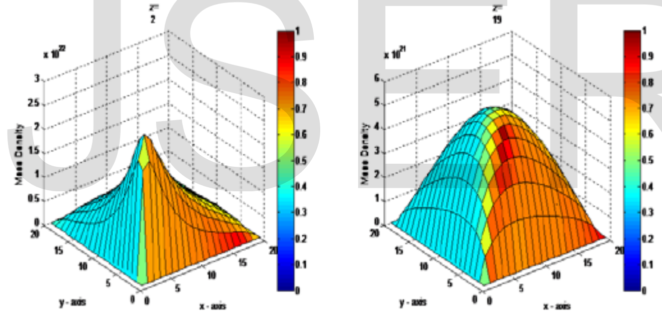
(b) Relation between velocity (km.s^{-1}) and space when n=20.



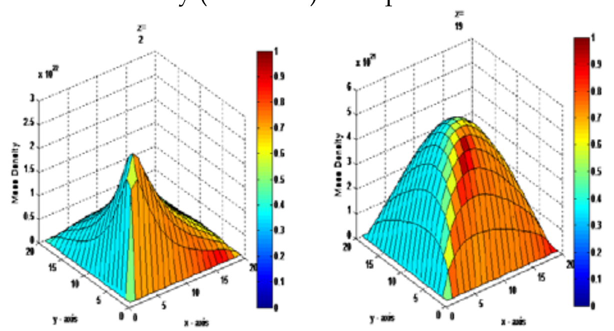
(c) Relation between velocity (km.s^{-1}) and space when n=30.
Figure(6) Relation between velocity (km.s^{-1}) of Hale Bopp comet and space (n=10,20, 30), using leapfrog method.



(a) relation between mass density (amu.cm^{-3}) and space at iteration of time when dt=0.1.



(b) Relation between mass density (amu.cm^{-3}) and space at iteration of time when dt =0.01.



(c) Relation between mass density (amu.cm^{-3}) and space at iteration of time when dt =0.001.
Figure(7) Relation between mass density (amu.cm^{-3}) of Hale Bopp and space at time iteration (dt =0.1, 0.01, 0.001).

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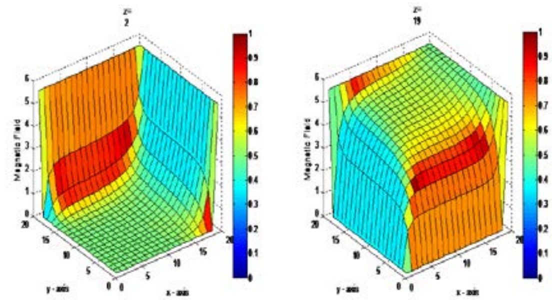
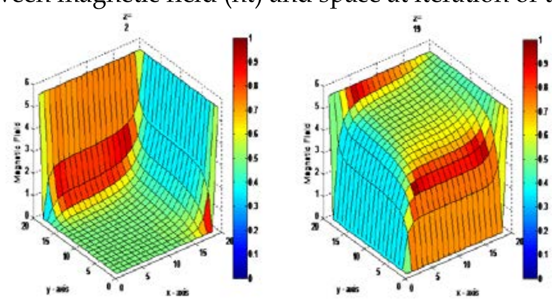
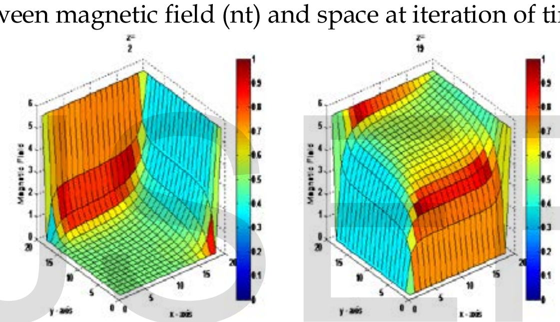


(a) Relation between magnetic field (nt) and space at iteration of time when δt =0.1.

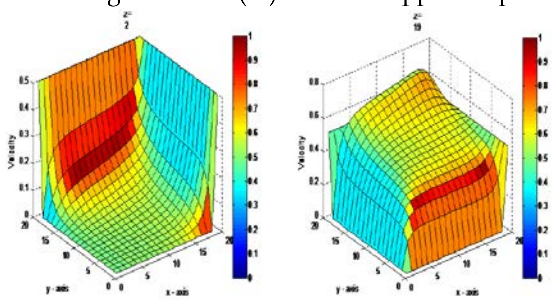


(b) Relation between magnetic field (nt) and space at iteration of time when δt =0.01.

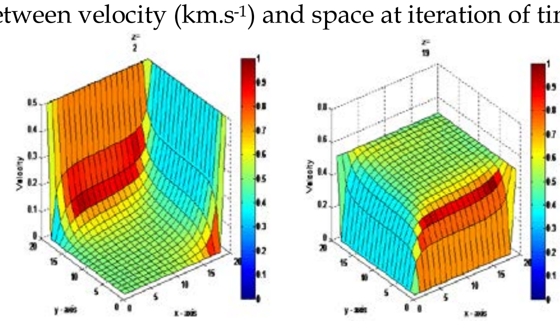


(c) Relation between magnetic field (nt) and space at iteration of time when δt =0.001.

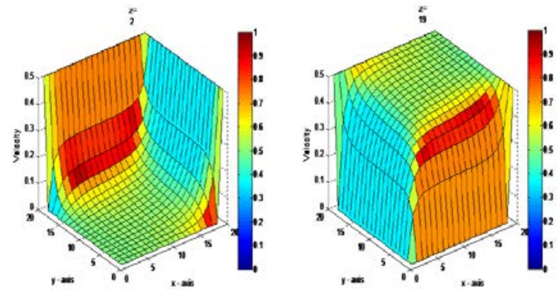
Figure(8) Relation between magnetic field (nt) of Hale Bopp and space at (δt =0.1, 0.01, 0.001).



(a) Relation between velocity ($km.s^{-1}$) and space at iteration of time when dt =0.1.



(b) Relation between velocity ($km.s^{-1}$) and space at iteration of time when dt =0.01.



(c) Relation between velocity (km.s^{-1}) and space at iteration of time when dt =0.001.
Figure(9) Relation between velocity (km.s^{-1}) of Hale Bopp comet and space at dt =(0.1,0.01, 0.001).

The velocity of Hale Bopp comet with variable value of dt is shown in figure (9) The results of velocity is similar to results of mass density and magnetic field, where it non stable in figure (A) and it stable in figures (B) and (C).

4 Conclusions

The results show that the mid-point leapfrog method has a higher (second) order of accuracy and it simple, stabile and more accurate from other approximations. Therefore, it has been used in the present res When changing the comets type with different physical properties it has been noted that results have been obtained from these comets are slightly changed in B and V results while the results of ρ changed more rapidly. When the spatial coordinate has been increased, it is seen that part of the information about the solar winds – comets interaction will be lost; this is due to the fact that increasing the mesh step will lead to improper solution MHD equation. When increasing the time coordinates it has been noted that more details about the interaction between the comets and solar wind can be obtained.

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